MATH 4650 / MSSC 5650 - Theory of Optimization

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Lectures: TTh 2-3:15p, Cudahy Hall 118 Course Website: D2L https://d2l.mu.edu/d2l/home/589381 Office Hours: TTh 1-2pm in-person, Cudahy Hall 367; or by appointment (e-mail me)

Course Description

Introduction to the theory of numerical optimization with applications. This course will cover fundamentals of nonlinear optimization: optimality conditions, gradient methods, Newton and quasi-Newton methods, and basics of constrained optimization and convex optimization. A special focus will be given to applications arising in data science, machine learning, and image/signal processing.

Course Objectives

Upon successful completion of the course, the student should be able to:

- 1. Give definitions of key mathematical terms (e.g., objective function, global and local minimizers, convexity, etc).
- 2. Classify an optimization problem according to whether it is constrained/unconstrained, convex/nonconvex, and identify a suitable algorithm to solve the optimization problem.
- 3. Understand first- and second-order optimality conditions for unconstrained smooth optimization of functions defined over ℝⁿ.
- 4. Understand basic convergence theory for selected algorithms.
- 5. Implement basic optimization algorithms in MATLAB/Python.
- 6. Apply existing solvers (e.g., CVX, scipy.optimize) to numerically solve optimization problems.

Textbook

Required:

• *Introduction to Nonlinear Optimization: Theory, Algorithms, and Applications with MATLAB* <u>2nd Edition</u>, by Amir Beck. SIAM, 2023.

I will also occasionally post additional notes and supplementary materials on D2L. You are not responsible for textbook material or any other material that is not covered in lecture.

Assessments

Grading Scale

Grades will be based on homework (50% of final grade), quizzes (15% of final grade), an in-class midterm exam (15% of final grade), and a take-home final exam (20% of final grade).

Letter grades for students in MATH 4650 will be assigned using the following scale:

Α	93–100%	C	73–76.99%
A-	90–92.99%	C-	70–72.99%
B+	87-89.99%	D+	67-69.99%
В	83-86.99%	D	60–66.99%
B-	80-82.99%	F	0–59.99%
C+	77–79.99%		

The same scale is used to assign letter grades for students in MSSC 5650, except the lowest non-failing grade is a *C* whose range is expanded to 70 - 76.99%.

Homework

- Homework will be assigned every 1-2 weeks, for a total of \sim 8 HW assignments (subject to change).
- Each homework assignment will be worth 25 points, and will consist of a mix of textbook problems and coding exercises.
- Some homework problems will be labelled [MSSC] and only the students enrolled in MSSC 5650 are required to do these problems. Students in Math 4650 may attempt these problems for extra credit.
- All homework will be submitted virtually to a D2L dropbox. Your homework must be uploaded as a single pdf file. There are several free apps to help with this, including Genius Scan and CamScanner among others. Please make sure your scanned homework is legible before uploading. Scanned homework that cannot be read or that is uploaded as multiple files and/or in the wrong format (i.e., not a single pdf) will not be graded and given an automatic 0. Occasionally, you may need to merge pdfs before uploading to D2L. There are several free online apps to do this as well, such as combinepdf.com.
- **Collaboration Policy**: It can be very helpful to study and work with a group. This type of cooperative learning is encouraged. However, be sure that you have a thorough understanding of the concepts as well as the steps used to solve an exercise. You must be able to work through the exercises on your own. Each student must write up their assignment individually and independently and must turn in their own work.
- External Resources/AI Policy: It is acceptable to consult external resources, including the internet and generative AI tools like ChatGPT, to get a better high-level understanding of topics in this course (e.g., looking at the wikipedia page for convex optimization, or prompting ChatGPT with "example of how to use fminbnd command in MATLAB"). It is not acceptable to copy solutions or code from these external resources on homework assignments, and doing so will be considered plagiarism. If you do use external resources, you are required to <u>cite all external resources</u> used to complete your assignment; failure to do so will be considered plagiarism.

If I suspect you are not the substantive author of parts of the assignment you turn in, and thus do not understand the material, you will be called in for an oral review. If the oral review is not passed to my satisfaction, then you will receive a zero on the assignment.

Finally, if you are unsure about whether it is ok to use an insight obtained from an external resource on the homework (e.g., a linear algebra fact not covered in lecture) **ask me in advance of turning in your assignment**.

• Late Work Policy: No late work will be accepted. However, your single lowest homework score will be dropped at the end of the semester.

Quizzes

- At the end of each class period, I will assign a D2L quiz consisting of one short answer question that will be due by the end of the following day.
- Each quiz is pass/fail.
- Your two lowest quiz scores will be dropped at the end of the semester.

Exams

- There will be one midterm exam and a comprehensive final.
- The midterm exam will be in-person and taken during regular class period. If you are unable to attend class on the day of the midterm due to quarantine/illness, please let me know and I can make accommodations.
- The final exam will be a take-home exam and due by upload to a D2L dropbox. You will have roughly one week to complete the exam.
- Make-up exams will not be given unless the student informs, and has a come to a written agreement with, the instructor regarding the absence no later than the day before the exam day. The student is responsible for scheduling their make-up exam.

Course Technology

MATLAB

- Several assignments will use the MATLAB computing toolbox. However, no previous experience with MATLAB is expected or required.
- There are several ways to use MATLAB. Marquette University students may download and install MATLAB onto their personal computers for free: https://techsquad.mu.edu/support/solutions/articles/21001160044. An online version of MATLAB that works in the browser is also available at: https://techsquad.mu.edu/support/solutions/articles/21001160044. An online version of MATLAB that works in the browser is also available at: https://techsquad.mu.edu/support/solutions/articles/21001160044. An online version of MATLAB that works in the browser is also available at: https://matlab.mathworks.com/. Finally, MATLAB is also installed on computers in the computer labs of Cudahy Hall and in Engineering and the library.
- Some homework assignments will make use of the **Image Processing Toolbox**. To add this toolbox to your MATLAB installation, click on the "APPS" tab at the top of the interface, then click the "Get More Apps" button, and search for "Image Processing Toolbox".
- Ensure you have access to MATLAB by the end of the first week of classes at the latest. Please contact ITS with questions about downloading and installing MATLAB on your device.

LaTeX

• LaTeX (pronounced "lay-tech" or "law-tech") is a text editor that enables you to create professionallooking mathematical documents. It is very commonly used in mathematics, computer science, physics, engineering, and other STEM fields. I highly recommend typesetting your homework in LaTeX, though it is not required. Overleaf (https://www.overleaf.com/) is a free, easy-to-use, online LaTeX editor (and is, in fact, what I used to create this document).

- I have posted some tutorial information and templates on D2L to help you get started, and I'm always more than happy to help out in office hours.
- Along with each homework assignment, I will share the LaTeX source on Overleaf that you can use to start from, so you don't need to retype the problem statements.

Course Policies

Class Conduct

Norms for classroom conduct are based on respect for the instructor and your fellow students. While in class, please silence your cell phones. Behaviors such as eating, sleeping, watching videos, or otherwise distracting your fellow students are inappropriate.

Attendance

Students are expected to attend the in-person lectures. However, I realize that this may not be possible for all students at all times. If a class is missed, the student is responsible for getting lecture notes from a classmate. In the event of a prolonged absence due to illness or other exceptional circumstances, please contact me as soon as possible so that I can make reasonable accommodations, especially if the absence includes an exam date.

Grading Disputes

If you have any issue with the grading of your homework or in-class exams you must bring it to my attention within seven days of the day the assignments were returned to the class; otherwise I cannot promise that I will consider the issue.

Accommodations for Disabilities

If you have a disability and will require accommodations under the Americans with Disabilities Act, you need to provide appropriate documentation to the Office of Disability Services. They will supply you with a letter to give to me detailing the accommodations. If you are unsure of whether or not you qualify for services, visit the Office of Disability Services' website, http://www.marquette.edu/disability-services, or contact them at (414) 288-1645. If you qualify for special accommodations you must work with the course instructor and come to an agreement no less than 7 days prior to the needed accommodation.

Academic Support

It is your responsibility to keep abreast of the course, to master the material covered, and to take the initiative for getting any help you may need. *You are encouraged to obtain help from the course instructor by attending office hours*. If you need additional support outside of class and office hours, the Office of Student Education Services (http://www.mu.edu/oses) is available to help.

Academic Integrity and Honesty

Academic dishonesty will not be tolerated. In particular, representing another person's work or the ouput of a generative AI model as your own is academic dishonesty. This applies to all homework, project work, assignments, take-home exams, etc. Any time you use and present ideas that are not your own you must cite your sources. Failure to abide by the university "Academic Integrity Policy" (http://bulletin.marquette.edu/undergrad/academicregulations/) may result in disciplinary action.

Course Schedule

Tues, Jan 14	Lecture 1 - Syllabus; Intro to Optimization		
Thurs, Jan 16	Lecture 2 - 1D Optimization		
Tues, Jan 21	Lecture 3 - Linear Algebra Review (Beck, Ch. 1)		
Thurs, Jan 23	Lecture 4 - Linear Algebra Review, Cont.		
Tues, Jan 28	Lecture 5 - Calculus Review (Beck, Ch. 1)		
Thurs, Jan 30	Lecture 6 - Calculus Review, Cont.		
Tues, Feb 4	Lecture 7 - Optimality Conditions (Beck, Ch. 2)		
Thurs, Feb 6	Lecture 8 - Optimality Conditions, Cont.		
Tues, Feb 11	Lecture 9 - Optimality Conditions, Cont.		
Thurs, Feb 13	Lecture 10 - Optimality Conditions, Cont.		
Tues, Feb 18	Lecture 11 - Least Squares and Applications (Beck, Ch 3)		
Thurs, Feb 20	Lecture 12 - Least Squares and Applications, Cont.		
Tues, Feb 25	Lecture 13 - The Gradient Method (Beck, Ch. 4)		
Thurs, Feb 27	Lecture 14 - The Gradient Method, Cont.		
Tues, Mar 4	Midterm Review		
Thurs Mar 6	In-class Midterm Exam		
Tues, Mar 11	No class - Spring Break		
Thurs, Mar 13	No class - Spring Break		
Tues, Mar 18	Lecture 15 - The Gradient Method, Cont.		
Thurs, Mar 20	Lecture 16 - Applications of the Gradient Method		
Tues, Mar 25	Lecture 17 - Applications of the Gradient Method, Cont.		
Thurs, Mar 27	Lecture 18 - Extensions of the Gradient Method, Cont.		
Tues, Apr 1	Lecture 19 - Newton's Method (Beck, Ch. 5)		
Thurs, Apr 3	3 Lecture 20 - Newton's Method, Cont. (Beck, Ch. 5)		
Tues, Apr 8	Lecture 21 - Constrained Optimization (Supplemental)		
Thurs, Apr 10	Lecture 22 - Constrained Optimization, Cont.		
Tues, Apr 15	Lecture 23 - Constrained Optimization, Cont.		
Tues, Apr 17	No class – Easter Break		
Tues, Apr 22	Lecture 24 - Convex Optimization (Beck Ch. 6, 7, 8, 9)		
Thurs, Apr 24	Lecture 25 - Convex Optimization, Cont.		
Tues, Apr 29	Lecture 26 - Convex Optimization, Cont.		
Thurs, May 1	Lecture 27 - Convex Optimization, Cont. (take-home final released)		
Thurs, May 8	Take-home Final Exam due by upload to D2L, 11:59 pm.		

Note: The schedule below is tentative and subject to change.