MATH 4650 / MSSC 5650 – Theory of Optimization

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Lectures: MW 2-3:15p, Cudahy Hall 131

Course Website: D2L https://d2l.mu.edu/d2l/home/506400

Office Hours: Mon & Fri 10:30-11:30am in-person, Cudahy Hall 367

Tues 1-2pm virtual, Teams Meeting Link: https://bit.ly/3GL3MHV

Course Description

Introduction to the theory of numerical optimization with applications. This course will cover fundamentals of continuous optimization: optimality conditions, gradient methods, Newton and quasi-Newton methods, and basics of constrained optimization and convex optimization. A special focus will be given to applications arising in data science, machine learning, and imaging.

Course Objectives

Upon successful completion of the course, the student should be able to:

1. Give definitions of key mathematical terms (e.g., objective function, global and local minimizers, convexity, etc).
2. Classify a continuous optimization problem according to whether it is constrained/unconstrained, convex/nonconvex, and identify a suitable algorithm to solve the optimization problem.
3. Implement optimization algorithms in MATLAB.
4. Understand first- and second-order optimality conditions for unconstrained smooth optimization.
5. Understand basic convergence theory for selected algorithms.

Textbook

Required:

- *Introduction to Nonlinear Optimization: Theory, Algorithms, and Applications with MATLAB* by Amir Beck. SIAM, 2014.

I will also occasionally post additional notes and supplementary materials on D2L. You are not responsible for textbook material or any other material that is not covered in lecture.

Assessments

Grading Scale

Grades will be based on homework (50% of final grade), quizzes (15% of final grade), an in-class midterm exam (15% of final grade), and a take-home final exam (20% of final grade).

Letter grades will be assigned using the following scale:

A	$93\% \leq$ Percentage $\leq 100\%$	**C**	$73\% \leq$ Percentage $< 77\%$
A-	$90\% \leq$ Percentage $< 93\%$	**C-**	$70\% \leq$ Percentage $< 73\%$
B+	$87\% \leq$ Percentage $< 90\%$	**D+**	$67\% \leq$ Percentage $< 70\%$
B	$83\% \leq$ Percentage $< 87\%$	**D**	$60\% \leq$ Percentage $< 67\%$
B-	$80\% \leq$ Percentage $< 83\%$	**F**	Below 60% Percent
C+	$77\% \leq$ Percentage $< 80\%$		

Homework

- Homework will be assigned every 1-2 weeks, for a total of $\sim$ 8 HW assignments (subject to change).
- Each homework assignment will be worth 25 points, and will consist of a mix of textbook problems and MATLAB coding exercises.
- Some homework problems will be labelled [MSSC] and only the students enrolled in MSSC 5650 need to do these problems. Students in Math 4650 may attempt these problems for extra credit.
- All homework will be submitted virtually to a D2L dropbox. **Your homework must be uploaded as a single pdf file.** There are several free apps to help with this, including Genius Scan and CamScanner among others. Please make sure your scanned homework is legible before uploading. Scanned homework that cannot be read or that is uploaded as multiple files and/or in the wrong format (i.e., not a single pdf) will not be graded and given an automatic 0. Occasionally, you may need to merge pdfs before uploading to D2L. There are several free online apps to do this as well, such as combinepdf.com.
- **Collaboration Policy**: It can be very helpful to study and work with a group. This type of cooperative learning is encouraged. However, be sure that you have a thorough understanding of the concepts as well as the steps used to solve an exercise. You must be able to work through the exercises on your own. Each student must write up their assignment individually and independently and must turn in their own work.
- It is acceptable to consult external resources (like the internet) while doing your homework. **It is not acceptable to copy large chunks of math or code from these external resources.** Solutions copied verbatim from Math StackExchange or similar forums will receive an automatic zero. **You are required to list all external resources used to complete your assignment. This includes names of any classmates you worked with. Failure to do so may be considered plagiarism.**
- No late work will be accepted. However, **your single lowest homework score will be dropped at the end of the semester**.

Quizzes

- At the end of each class period, I will assign a D2L quiz consisting of one short answer question that will be due by the end of the following day.
- Each quiz is pass/fail.
- Your three lowest quiz scores will be dropped at the end of the semester.

Exams

- There will be one midterm exam and a comprehensive final.
- The midterm exam will be in-person and taken during regular class period. If you are unable to attend class on the day of the midterm due to quarantine/illness, please let me know and I can make accommodations.
- The final exam will be a take-home exam and due by upload to a D2L dropbox. You will have roughly one week to complete the exam.
- Make-up exams will not be given unless the student informs, and has a come to a written agreement with, the instructor regarding the absence no later than the day before the exam day. The student is responsible for scheduling their make-up exam.

Course Technology

MATLAB

- Several assignments will use the MATLAB computing toolbox. However, no previous experience with MATLAB is expected or required.
- Marquette University students may download MATLAB onto their personal computers: http://www.marquette.edu/its/help/matlab/. MATLAB is also available in the computer labs of Cudahy Hall and in Engineering and the library.
- Some homework assignments will make use of the **Image Processing Toolbox**. To add this toolbox to your MATLAB installation, click on the "APPS" tab at the top of the interface, then click the "Get More Apps" button, and search for "Image Processing Toolbox".
- You are encouraged to check access to MATLAB by Friday Jan 20 to ensure proper access moving forward. Please contact ITS with questions about downloading and installing MATLAB on your device.

LaTeX

- LaTeX (pronounced "lay-tech" or "law-tech") is a text editor that enables you to create professional-looking mathematical documents. It is very commonly used in mathematics, computer science, physics, engineering, and other STEM fields. **I highly recommend writing up your homework and/or the take-home exam in LaTeX.** Overleaf (https://www.overleaf.com/) is a free, easy-to-use, online LaTeX editor (and is, in fact, what I used to create this document).
- I have posted some tutorial information and templates on D2L to help you get started, and I'm always more than happy to help out in office hours.

Course Policies

Class Conduct

Norms for classroom conduct are based on respect for the instructor and your fellow students. While in class, please silence your cell phones. Behaviors such as eating, sleeping, watching videos, or otherwise distracting your fellow students are inappropriate.

Grading Disputes

If you have any issue with the grading of your homework or in-class exams you must bring it to my attention within seven days of the day the assignments were returned to the class; otherwise I cannot promise that I will consider the issue.

Accommodations for Disabilities

If you have a disability and will require accommodations under the Americans with Disabilities Act, you need to provide appropriate documentation to the Office of Disability Services. They will supply you with a letter to give to me detailing the accommodations. If you are unsure of whether or not you qualify for services, visit the Office of Disability Services' website, http://www.marquette.edu/disability-services, or contact them at (414) 288-1645. If you qualify for special accommodations you must work with the course instructor and come to an agreement no less than 7 days prior to the needed accommodation.

Academic Support

It is your responsibility to keep abreast of the course, to master the material covered, and to take the initiative for getting any help you may need. *You are encouraged to obtain help from the course instructor by attending office hours.* If you need additional support outside of class and office hours, the Office of Student Education Services (http://www.mu.edu/oses) is available to help.

Academic Integrity and Honesty

Academic dishonesty will not be tolerated. In particular, representing another person's work as your own is academic dishonesty. This applies to all homework, project work, assignments, take-home exams, etc. Any time you use and present ideas that are not your own you must cite your sources. Failure to abide by the university "Academic Integrity Policy"(http://bulletin.marquette.edu/undergrad/academicregulations/) may result in disciplinary action.

Tentative Course Schedule

Monday	Wednesday
Jan 16th No Class	18th **1** **Unit 1: 1D Optimization** Intro to Optimization
23rd **2** Optimality conditions in 1D	25th **3** Steepest Descent & Newton's Method in 1D
30th **4** **Unit 2: Linear Algebra and Calculus Review** Linear Algebra Review	Feb 1st **5** The Spectral Theorem PCA & Eigenfaces
6th **6** Topology of $\mathbb{R}^n$ Limits and Continity	8th **7** Derivatives and Taylor Series
13th **8** **Unit 3: Optimality Conditions** Global vs. Local Optima Necessary 1st order conditions	15th **9** Classification of symmetric matrices 2nd order conditions
20th **10** 2nd order conditions, Cont. Gloabl optimality	22nd **11** **Unit 4: Least Squares** The Normal Equations
27th **12** Data Fitting & Linear Regression Regularized Least Squares	Mar 1st **13** Image Processing Applications of Regularized Least Sqaures
6th **14** Midterm Review	8th In-class Midterm Exam
13th No Class	15th No Class
20th **15** **Unit 5: Gradient Methods** Descent Direction Methods	22nd **16** Gradient Descent
27th **17** Steepest Descent	29th **18** Applications: Logistic Regression, Image Restorataion
Apr 3rd **19** Convergence analysis of Gradient Descent, Part I	5th **20** Convergence analysis of Gradient Descent, Part II
10th No Class	12th **21** Momentum Methods
17th **22** Stochastic Gradient Descent	19th **23** Application: Training Neural Networks
24th **24** **Unit 6: Extensions** Newton's Method	26th **25** Constrained Optimization
May 1st **26** Convex Optimization	3rd **27** Proximal Gradient Descent
8th	10th **28** Final Take-home Exam Due